## ENTRANCE EXAMINATION IN MATHEMATICS AND INFORMATICS FOR APPLICANTS ENTERING MASTER'S PROGRAMS OF PHYSTECH SCHOOL OF APPLIED MATHEMATICS AND COMPUTER SCIENCE

## Regulations

The admission process includes three main steps: a programming contest, a written exam and an oral exam.

The programming contest consists of 4-7 problems which are evaluated and graded automatically. The contest duration is 3 hours (180 minutes).

The written exam consists of 6-10 problems of different complexity. All problems require a complete solution with a detailed proof and/or explanation. It is legal to use the following online resources during the written exam:

- Wikipedia.org
- Wolframalpha.com
- Live.sympy.org
- Python.org/shell

Computational results based on one of these systems are not accepted as a complete solution or a proof. These resources can only be used to get a hint or recap some main facts and properties. The written exam duration is 3 hours (180 minutes).

The oral exam includes discussion of applicant's experience and motivation, theoretical questions and may include questions on the provided written exam solutions.

Programming contest program

- 1. Data types and basic operations in Python
- 2. Data structures in Python
- 3. Linear operations and computations with matrices
- 4. Random numbers
- 5. Basic data processing

Written and oral exams program

- 1. Sequences. Limits of sequences. Examples of convergent and divergent sequences.
- 2. Continuous functions of one variable. Limits of functions.
- 3. Derivative. Differentiable functions. Mean value theorems: Fermat, Roll, Lagrange, Cauchy.
- 4. Infinitely small and limited quantities. Big-O notation.
- 5. Taylor series.
- 6. Indefinite integrals. Antiderivative.
- 7. Definite integrals.
- 8. Newton Leibnitz theorem.

- 9. Multivariate calculus.
- 10. Gradient. Jacobian matrix.
- 11. Systems of linear equations and the Gaussian elimination.
- 12. Vector spaces. Definition, examples: a space of rows, spaces of square matrices, spaces of symmetric and skew-symmetric square matrices, spaces of polynomials of one variable.
- 13. Linearly independent and linearly dependent systems of vectors.
- 14. A basis and the dimension of a vector space.
- 15. Matrix determinant. Trace.
- 16. Inverse matrix.
- 17. Orthogonal matrices and unary operators.
- 18. Eigendecomposition. Eigenvectors and eigenvalues.
- 19. Elementary events and finite sample spaces. The classical definition of probability. Computation of probabilities in classical settings.
- 20. Standard counting rules: the rule of sum and the rule of product.
- 21. Combinations, placements and permutations.
- 22. Newton's binomial theorem.
- 23. Continuous and discrete random variables. CDF and PDF.
- 24. Mathematical expectation and variance

## References

- E.B. Vinberg, «A Course in Algebra», Graduate Studies in Mathematics, AMS, Vol. 56, 2003.
- 2. W. Rudin, «Principles of Mathematical Analysis», International Series in Pure and Applied Mathematics, McGraw-Hill Education, 1976, 3rd Edition.
- 3. L.B. Koralov, Ya.G. Sinai, «Theory of Probability and Random Processes», Springer-Verlag Berlin Heidelberg, 2007.
- Ian Goodfellow and Yoshua Bengio and Aaron Courville. "<u>Deep Learning</u>". MIT Press, 2016
- Marc Peter Deisenroth, A. Aldo Faisal, and Cheng Soon Ong. "<u>Mathematics for Machine</u> <u>Learning</u>". Cambridge University Press, 2020